

UNIT 6: WINDOWING

BASICDSP

6.1 Introduction

This unit considers the consequences of analysing sections or 'windows' of a continuous signal, and how the use of smoothing functions reduces the spectral artifacts introduced.

When you have worked through this unit you should:

- be able to state why and when spectral leakage occurs
- understand how a smooth window reduces the amount of spectral leakage
- be able to describe and apply a Hamming window in your own work

6.2 Concepts

Where the frequency components of signals we analyse with the DFT are exact harmonics of the DFT analysis, the resulting spectrum shows clean spectral lines. Since mostly the signals we want to analyse will not contain just these harmonics, we find that the energy at one input frequency is spread to a number of nearby harmonics in the DFT. This is called *spectral leakage*.

A similar problem arises for signals which are changing with time, where we usually want to analyse a 'snapshot' of the signal, short enough in which we could consider the generating system to be 'stationary'. To do this we must take a sample or a *window* of the signal over some defined interval. If we simply cut out a section of the signal, however, we introduce artefacts into the waveform - namely the sudden onset and offset - which will be manifested as distortions in our spectral analysis. What is happening is that our spectrum results from the convolution of the signal spectrum with the spectrum of a *rectangular* window. The spectrum of a rectangular window has the familiar $\sin(x)/x$ shape or *sinc* function, and so each signal component is broadened by this shape.

We can reduce these distortions by ensuring that the section has no sudden onset or offset, which we can do by multiplying our section with a smoothing function which reduces the size of the signal at the edges. We need a shape which has a spectrum with a narrow central lobe and small sidelobes. A window based on a *raised-cosine* shape called the *Hamming* window is a common compromise:

$$w[n] = 0.54 - 0.46 \cos\left(\frac{2\pi n}{N}\right)$$

Where N is the number of samples in the section. See `Hamming()`.

Algorithms

```
' Generate a Hamming Window
Public Shared Function Hamming(ByVal len As Integer) As Waveform
    Dim owv As New Waveform(len, 1)

    Dim omega As Double = 2.0 * Math.PI / (len - 1)

    For i As Integer = 1 To owv.Count
        owv(i) = (0.54 - 0.46 * Math.Cos(omega * (i - 1)))
    Next

    Return owv
End Function

' Applies a Hamming window to the given waveform segment
Public Shared Function Hamming(ByVal iwv As Waveform) As Waveform
    Dim owv As New Waveform(iwv.Count, iwv.Rate)

    Dim omega As Double = 2.0 * Math.PI / (iwv.Count() - 1)

    For i As Integer = 1 To owv.Count
        owv(i) = (0.54 - 0.46 * Math.Cos(omega * (i - 1))) * iwv(i)
    Next

    Return owv
End Function

' Applies a Hamming window to the given complex waveform segment
Public Shared Function Hamming(ByVal iwv As ComplexWaveform) As ComplexWaveform
    Dim owv As New ComplexWaveform(iwv.Count, iwv.Rate)

    Dim omega As Double = 2.0 * Math.PI / (iwv.Count() - 1)

    For i As Integer = 1 To owv.Count
        owv(i) = (0.54 - 0.46 * Math.Cos(omega * (i - 1))) * iwv(i)
    Next

    Return owv
End Function
```

Bibliography

Lynne & Fuerst Introductory Digital Signal Processing, Section 8.2.2

Example Program 6.1

```
Imports BasicDSP
Imports ZedGraph
Public Class TestWindow

    Private Sub TestWindow_Load(ByVal sender As System.Object, ByVal e As
System.EventArgs) Handles MyBase.Load

        ' get sample waveform
        Dim sig As New Signal("c:/sfs/demo/six.wav")
        Dim wv As Waveform = sig.Cut(14537, 1024).Float
        Dim cwv1 As ComplexWaveform = wv.Complex

        ' create the graphs
        Dim gp As New Graph(Me.CreateGraphics, zgc, 2, 2, "Effects of
Windowing")

        ' apply rectangular window and plot
        For i As Integer = 1 To cwv1.Count \ 4
            cwv1(i) = 0
        Next
        For i As Integer = 3 * cwv1.Count \ 4 To cwv1.Count
```

```

        cwv1(i) = 0
    Next
    gp.PlotComplexWaveform(1, cwv1, "Rectangular Window")

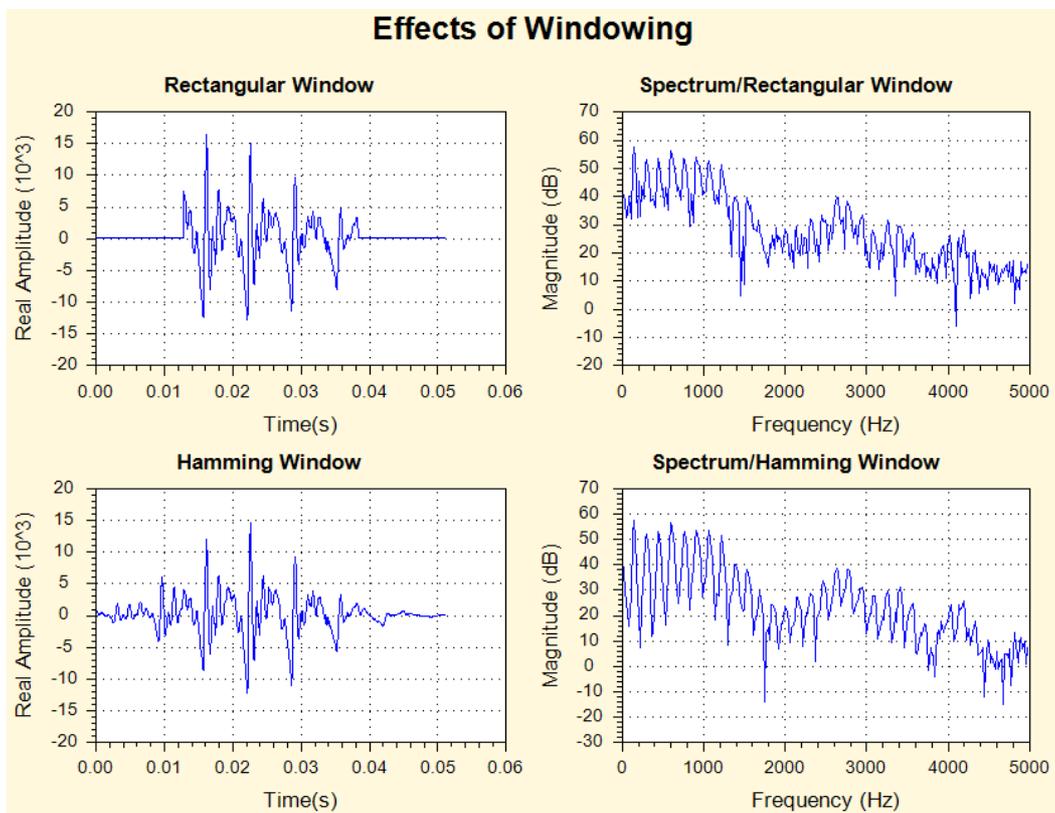
    ' FFT without windowing and plot
    Dim sp1 As Spectrum = DFT.ComplexFFT(cwv1)
    gp.PlotDbSpectrum(2, sp1.Half.Half, "Spectrum/Rectangular Window")

    ' apply Hamming window and plot
    Dim cwv2 As ComplexWaveform = Window.Hamming(wv.Complex)
    gp.PlotComplexWaveform(3, cwv2, "Hamming Window")

    ' FFT with Hamming window and plot
    Dim sp2 As Spectrum = DFT.ComplexFFT(cwv2)
    gp.PlotDbSpectrum(4, sp2.Half.Half, "Spectrum/Hamming Window")

    End Sub
End Class

```



Exercise

- 6.1 Adapt Example 6.1 to plot the waveforms and FFT spectra of sine waves at $F_s/4$ (exact) and $F_s/6$ (non-exact) harmonic frequencies with rectangular and Hamming windows. Use a short FFT of about 128 samples. Interpret the results.